

# Hadronic Contributions to $R$ and $g - 2$ from Initial-State-Radiation Data

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## Abstract

I review the recent efforts to improve the precision of the prediction of the anomalous moment of the muon, in particular of the hadronic contribution of the vacuum polarization, which is the contribution with the largest uncertainty. Focus is given to the recent result for  $e^+e^- \rightarrow \pi^+\pi^-$  by the BaBar collaboration, obtained using events with radiation in the initial state.

## 1. Introduction

Elementary particles have a magnetic moment  $\vec{\mu}$  proportional to their spin  $\vec{s}$ , with  $\vec{\mu} = (ge)/(2m)\vec{s}$ . While pointlike Dirac particles would have  $g = 2$ , i.e. an “anomalous” relative deviation of  $a \equiv (g-2)/2 = 0$ , Nafe *et al.* observed the first hints of a significant deviation from  $a_e = 0$  more than 60 years ago [1]. The following year, Schwinger computed [2] the first-order contribution to  $a$ , equal to  $\alpha/(2\pi)$ , the diagram for which is shown in Fig. 1.

The development of the quantum electro-dynamics (QED) followed, and later of gauge theories in general, making these early works the very basis of our present understanding of the elementary world. Tremendous efforts have been devoted to improving the precision of the theoretical prediction and of the direct measurement of  $a$  since then [3].

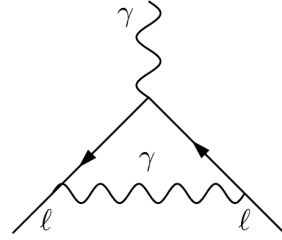
More than 60 years later the situation is pretty exciting, with the experimental and theoretical precision on the anomalous magnetic moment of the muon  $a_\mu$  both of the order of  $6 \times 10^{-10}$ , and a discrepancy of  $(29 \pm 9) \times 10^{-10}$  between them, i.e. amounting to  $3.2 \sigma$ , should Gaussian statistics be assumed (Table 1.).

**Table 1.** Summary of the contribution to the theory prediction of the value of  $a_\mu$ , compared with the experimental measurement [3].

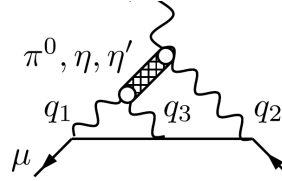
QED	116 584 71.81	$\pm 0.02$
Leading hadronic VP	690.30	$\pm 5.26$
Sub-leading hadronic VP	-10.03	$\pm 0.11$
Hadronic light-by-light	11.60	$\pm 3.90$
Weak (incl. 2-loops)	15.32	$\pm 0.18$
Theory	11659179.00	$\pm 6.46$
Experiment [4]	11659208.00	$\pm 6.30$
Exp - theory	29.00	$\pm 9.03$

The largest contribution to  $a_\mu$  is by far from QED but its contribution to the uncertainty is negligible. In terms of uncertainty, the main contribution is from the hadronic component of the one-loop vacuum polarization (VP, Fig. 2.) and, to a lesser extent, from the hadronic component of the light-by-light processes (Fig. 3.).

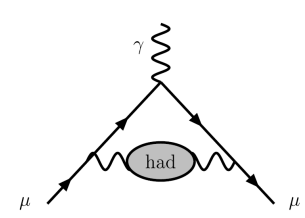
The photon propagator with VP is obtained from the bare propagator by replacing the electric charge  $e$  by the



**Fig. 1.** 1st order contribution to  $a$ .



**Fig. 3.** A light-by-light diagram.



**Fig. 2.** Lowest-order hadronic VP diagram.

energy dependant quantity:

$$e^2 \rightarrow e^2/[1 + (\Pi'(k^2) - \Pi'(0))],$$

where  $k$  is the photon 4-momentum. At low energy, hadronic processes are not computable with the desired precision. Instead the VP amplitude  $\Pi(k^2)$  is obtained from the dispersion relation:

$$\Pi'(k^2) - \Pi'(0) = \frac{k^2}{\pi} \int_0^\infty \frac{Im\Pi'(s)}{s(s - k^2 - i\epsilon)} ds,$$

which in turn is related through the optical theorem:

$$Im\Pi'(s) = \alpha(s)R_{\text{had}}(s)/3,$$

to the ratio:

$$R_{\text{had}}(s) = \sigma_{\text{had}} \frac{3s}{4\pi\alpha(s)} = \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}.$$

Finally, the hadronic VP contribution is obtained from the “dispersion integral”:

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R_{\text{had}}(s)\hat{K}(s)}{s^2} ds,$$

where  $\hat{K}(s)$  is an analytical function that takes values close to 1. We note, from the  $1/s^2$  variation of the integrand that the dominant contribution comes from the low energy part of the integral. A good experimental

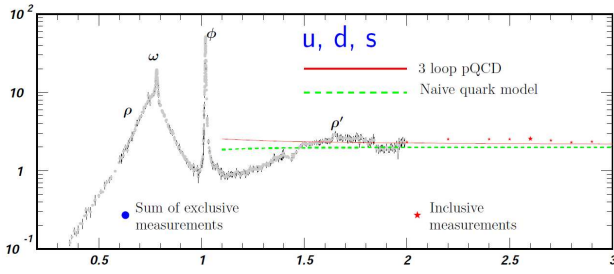


Fig. 4.  $R_{\text{had}}$  as a function of  $\sqrt{s}$  (GeV) [5].

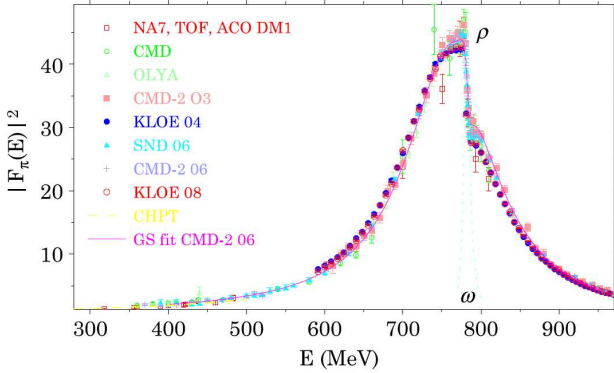


Fig. 5. Variation of the pion form factor squared with energy [3] (KLOE 04 is superseded by KLOE 08).

precision of the measurement of  $R_{\text{had}}(s)$  at low energy is therefore welcome. Fig. 4. shows a summary of the present measurements of  $R_{\text{had}}(s)$  [5], where the presence of  $J^{PC} = 1^{--}$  mesons can be seen.

The  $\pi^+\pi^-$  channel has both the largest contribution and dominates the uncertainty, with  $a_{\mu}^{\pi^+\pi^-}[2m_{\pi}, 1.8\text{GeV}/c^2] = (504.6 \pm 3.1(\text{exp}) \pm 0.9(\text{rad})) \times 10^{-10}$ , compared to the full  $a_{\mu}^{\text{had}} = (690.9 \pm 5.3) \times 10^{-10}$  from Table 1.. A summary of direct measurements in terms of the squared pion form factor is shown in Fig. 5.. The  $3.2\sigma$  discrepancy mentioned above is computed using this  $\pi\pi$  input.

## 2. $\tau$ decay spectral functions

The  $I = 1$  part of the  $e^+e^- \rightarrow \pi^+\pi^-$  cross-section can be estimated from the spectral function of  $\tau$  decays to  $\nu\pi^+\pi^0$ , under the hypothesis of conservation of the weak vector current (CVC). The method was pioneered by the ALEPH collaboration [6] and followed by OPAL [7] and CLEO [8]. Recently the Belle collaboration has performed an analysis using much larger statistics [9], obtaining a value compatible with, and more precise than, the combination of all previous results [10].

The  $\tau$  method provides a high experimental precision, but extracting the contribution to  $a_{\mu}$  depends on making a number of isospin-breaking (IB) corrections. A recent update [11] of Ref. [10] lowers the correction by  $\approx 7 \times 10^{-10}$ , while the uncertainty on the correction is now  $1.5 \times 10^{-10}$ .

The branching fraction of the  $\tau \rightarrow \nu\pi^+\pi^0$  decay also takes part in the calculation, with a 0.5 % uncertainty.

## 3. $e^+e^- \rightarrow \pi^+\pi^-$ using ISR method

Initial-state radiation makes it possible to measure the cross-section of the production of a final state  $f$  in  $e^+e^-$  collisions at a squared energy  $s$ , over a wide range of energies, lower than  $\sqrt{s}$ , through the radiation of a high

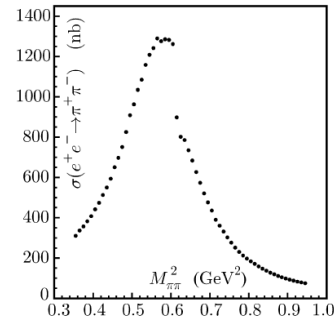


Fig. 6.  $e^+e^- \rightarrow \pi^+\pi^-$  cross section measured by KLOE, using the ISR method [15].

energy photon by one of the incoming electrons, after which the electrons collide at a squared energy  $s'$ .

Recently the BaBar experiment has developed a systematic program of measurements of cross-sections of  $e^+e^-$  to hadrons at low energy, using the ISR method [14]. The boost undergone by the final state  $f$  provides an excellent efficiency down to threshold. In all studies by BaBar, the ISR photon is observed ( $\gamma$ -tag) and its direction is compared to the direction predicted from the direction of  $f$ , providing a powerful background noise rejection. Most of these measurements are more precise than the previously available results by about a factor of three.

### 3.1. KLOE's result on $e^+e^- \rightarrow \pi^+\pi^-$

The KLOE experiment, when running on the  $\phi$  resonance, has studied the  $e^+e^-$  annihilations to  $\pi^+\pi^-$  with the ISR method [15], Fig. 6.. Here the ISR photon is not reconstructed: the request that the photon direction be compatible with its having been emitted in the beam pipe allows mitigation of the background to some extent, but the systematical uncertainty on background subtraction is still a major component of the total uncertainty. The radiator function is provided from simulation, with systematics of 0.5%, is the other major component.

The value of  $a_{\mu}^{\pi^+\pi^-}$  obtained is compatible with the combination of previous results by CMD-2 & SND over the mass range that they have in common of (630 - 958 MeV/ $c^2$ ).

### 3.2. BaBar's result on $e^+e^- \rightarrow \pi^+\pi^-$

BaBar uses a different approach: photon tagging is used, and the ISR luminosity is obtained from the muon channel,  $e^+e^- \rightarrow \mu^+\mu^-\gamma$  [16]. The systematics related to additional radiation is minimized in this NLO measurement, i.e. radiation of one possible additional photon is allowed, so that the final states actually reconstructed are  $\pi^+\pi^-\gamma(\gamma)$  and  $\mu^+\mu^-\gamma(\gamma)$ . The “bare” ratio  $R_{\text{had}}(s')$  mentioned above is obtained from the experimentally measured  $R_{\text{exp}}(s')$  after correction of final state radiation (FSR) in  $e^+e^- \rightarrow \mu^+\mu^-$  and of additional FSR in ISR events  $e^+e^- \rightarrow \mu^+\mu^-\gamma$ .

A number of important systematics cancel when measuring the  $\pi/\mu$  ratio, such as those associated with the collider luminosity, the efficiency of the reconstruction of the ISR photon, and the understanding of additional ISR radiation.

The limiting factor is then the understanding of the possible “double”  $\pi - \mu$ , MC-data efficiency discrepan-

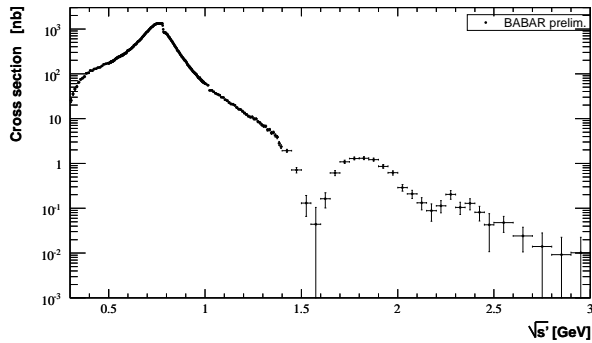


Fig. 7. Bare, unfolded  $e^+e^- \rightarrow \pi^+\pi^-$  cross section measured by BaBar, using the ISR method [16].

cies. These are studied in detail, with methods designed to disentangle correlations as much as possible. For example, inefficiency of the track-based trigger is studied using events selected with a calorimetry-based trigger – the small correlation between both triggers being studied separately. Likewise,  $\mu$  and  $\pi$  particle identification (PID) efficiency is studied in good-quality, two-tracks ISR events, in which either one, or both, tracks meet the PID selection criteria. Concerning tracking, a sizable degradation of the efficiency for tracks overlapping in the detector was observed and studied in detail.

The systematics finally obtained are of the order of, or smaller than, 1 % over the whole mass range studied; i.e., from threshold to 3 GeV/ $c^2$ .

The  $e^+e^- \rightarrow \pi^+\pi^-$  cross section measured by BaBar is shown in Fig. 7.. The sharp drop due to the interference between the  $\rho$  and the  $\omega$ , already present in Fig. 5, is clearly visible. The interference between the successive radial excitations of the  $\rho$  induces these dips in the cross-section.

The measured value of  $a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\text{GeV}/c^2] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$  has a precision similar to the combination of all previous  $e^+e^-$ -based results, but is larger by about  $2.0\sigma$ .

In addition to the measurement of the  $\pi/\mu$  ratio, and extraction of the  $e^+e^- \rightarrow \pi^+\pi^-$  cross section, BaBar has compared its  $\mu^+\mu^-$  spectrum to the Monte Carlo prediction, finding a good agreement within  $0.4 \pm 1.1\%$ , dominated by the collider luminosity uncertainty of  $\pm 0.9\%$ .

The distribution of the squared pion form-factor is fitted with a vector-dominance model including the resonances  $\rho, \rho', \rho'', \omega$ , with the  $\rho$ 's being described by the Gounaris-Sakurai model. The fit (Fig. 8.) yields a good  $\chi^2/n_{df}$  of 334/323, and parameters compatible with the world-average values. We can then use the fitted model to compare the result by BaBar with that of previous measurements (Fig. 9.). The BaBar result is a bit larger than that obtained by CMD2[18] and SND [19], nicely compatible with the high-statistics  $\tau$ -based result by Belle, but shows a clear disagreement with KLOE.

#### 4. $a_\mu$ : the present situation

The present situation in terms of  $a_\mu$  is summarized in Fig. 10.:

- The four upper points show that there is a general agreement between the various recent combinations of direct  $e^+e^-$ -based  $\pi^+\pi^-$  measurement [12, 13, 11].
- The large discrepancy between computations of  $a_\mu$  based on these and the experimental measurement

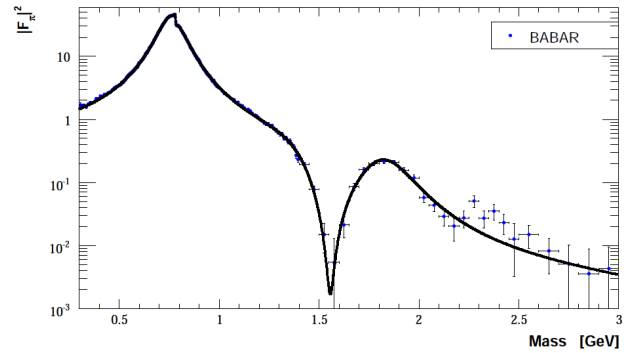


Fig. 8. VDM fit of the squared pion form-factor by BaBar [16].

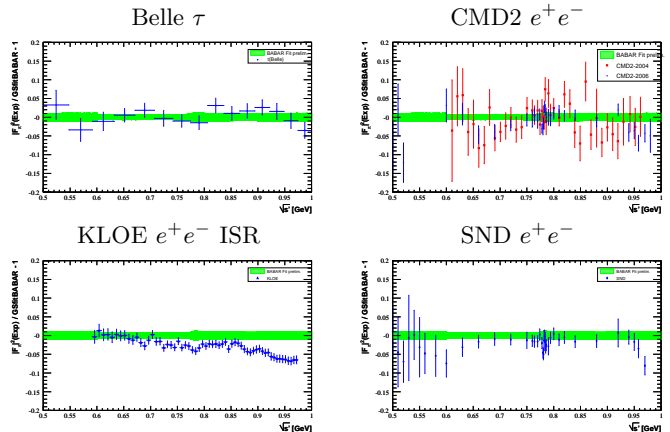


Fig. 9. Relative difference between the BaBar result, as fitted in Fig. 8., with that of previous experiments.

by BNL-E821 [4] is clear.

- The combination of  $\tau$ -based results, when corrected for isospin-breaking effects using the most recent calculation [11], are also significantly lower than the experimental measurement [4], by  $1.8\sigma$ .
- My computation of  $a_\mu$  using the BaBar  $\pi^+\pi^-$  measurement [16] only is larger than the combination of previous  $e^+e^-$ -based measurements, and compatible with the  $\tau$ -based result, but still  $2.4\sigma$  away from BNL-E821 [4].
- The combination of all  $e^+e^-$ -based measurements, including the recent one by BaBar, show an uncertainty that has decreased significantly, and a central value that is larger. However, the significance of the difference with respect to BNL-E821 [4] is barely changed, of the order of  $3.3\sigma$ .

After my talk, Changzheng Yuan mentioned a soon-to-appear preprint on “Reevaluation of the hadronic contribution to the muon magnetic anomaly using new cross section data from BABAR” that is now available [17], and that obtains (more carefully) similar results as that I have shown in Figure 10..

#### 5. What might take place in the next decade

##### 5.1. $a_\mu$ measurement

One single high-precision heavily-statistics-dominated experimental measurement of  $a_\mu$  [4] is facing a prediction in which the contribution with largest uncertainty

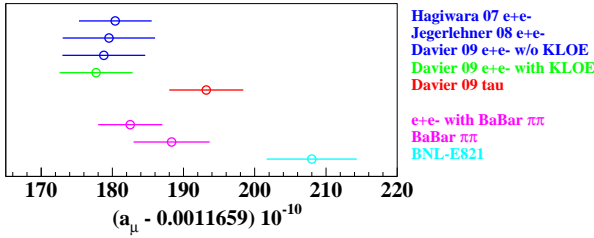


Fig. 10.  $a_\mu$ : the present situation.

has been confirmed, within reasonable significance, by a number of measurements using various methods, each affected by its own systematics.

The obvious next step, before calling for new physics, is therefore to check the measurement:

- A new collaboration is planning to move the experimental apparatus from BNL to FNAL, and perform a new measurement with statistics increased by a factor of 50, and reduced systematics, bringing the experimental uncertainty down to 0.14 ppm, i.e.,  $1.6 \times 10^{-10}$  [20].
- It would obviously be intensely desirable to cross-check such a measurement using a completely different set-up. An alternative scheme is explored at J-PARC, with a micro-emittance muon beam inside a high-precision magnetic field, mono-magnet storage “ring” [21].

## 5.2. Prediction

On the prediction side, the main effort is understandably devoted to the hadronic VP contribution.

- BaBar will complete its ISR program and provide measurements of all possible hadronic final states in the low energy range relevant to this discussion.
- Belle may check BaBar’s  $\pi^+\pi^-$  measurement and BaBar may check Belle’s  $\tau$  spectral functions. KLOE is working on an analysis with photon tagging too.
- BES III will measure  $R_{\text{had}}(s)$  in the range 2.0 – 4.6 GeV, something that will improve on  $a_\mu$  only marginally, but will also measure the  $\tau \rightarrow \nu\pi^+\pi^0$  branching fraction with improved precision [22], an important ingredient in the use of the  $\tau$ -based spectral functions.

The recent calculation of isospin-breaking corrections [11] will be published and will doubtlessly be cross-checked by other authors.

- The collider at Novosibirsk has been upgraded to VEPP-2000 [23], and the CMD [24] and SND experiments too.

Following the vacuum polarization, the next target in line for improvement is the contribution of light-by-light scattering. Here too work is in progress and there is hope to improve the precision, both theoretically [25], and using results of the  $\gamma\gamma$  programme at  $DA\Phi NE - 2$  [26]

In total, there is good hope to bring both the prediction and experimental uncertainties of  $a_\mu$  at a very few  $10^{-10}$ .

## 6. Acknowledgements

I’d like to thank the organizers of PIC2009 for having invited me to present this review and for having set the scene for such an interesting gathering on the most recent results in particle physics and high energy astronomy.

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